What Are The Effects of Fiscal Policy Shocks in India?*

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*The views expressed herein are those of the author(s) and should not be attributed to the IMF, its Executive Board, or its management.

Abstract

1. Introduction

In response to the largest economic downturn since the 1930s, several countries around the world implemented large fiscal stimulus to cushion the blow from the financial crisis and

jump start the economic recovery. During the initial phases of the crisis, policy makers concerns about the effectiveness of monetary policy, stemming from very low interest rates to weak transmission mechanism, led to embark in sizable fiscal stimuli packages to offset falling private sector demand. India was no exception to this. Despite the much shallower slowdown in overall economic activity, industrial production growth fell markedly and overall financing conditions tightened significantly during the acute phase of the crisis. The Indian authorities undertook several measures to address the economic fallout from the crisis. On the fiscal front, the Indian government implemented large expansionary measures in 208/09 and 2009/10. As a result of the fiscal expansion, the deficit increased sharply and the contribution of government consumption to GDP growth in the last two quarters of 2008/09 was sizable. This paper assesses the effectiveness of fiscal policy in India.





Even as large fiscal stimuli packages are being implemented around the world, the effectiveness of fiscal policy to counter falling aggregate demand has been called increasingly into question. In particular, the evidence on the magnitude of fiscal multipliers has become a hotly debated issue in academic as well as policy circles. Unfortunately, theoretical models yield wide ranges of fiscal multipliers depending on assumptions about the functioning of the economy (e.g., degree of price rigidity) and structural parameters (labor supply elasticity), and to complicate matters further, empirical estimates of the impacts of fiscal policy also vary significantly and are highly dependent on the methodology employed (Perotti, 2009).¹ Nonetheless, as the Indian authorities have started to exit from

¹ In addition to the academic literature spearheaded by Blanchard and Perotti (2002), since the release of the current U.S. administration's assumptions about the effects of fiscal expansion on (continued)

accommodative stance in a calibrated way, having estimates of fiscal multipliers is likely to be useful. In addition, shedding light on the size of fiscal multipliers could also enhance our understanding of other features of the economy and help assess the extent of crowding-out going forward.

This paper analyzes empirically the effectiveness of fiscal policy in India. We apply simple structural and recursive vector autoregression (VAR) to gauge the effects of fiscal policy on GDP and other macroeconomic variables. The data used span the period 1996-2009, covering a period of mild deficits and the fiscal consolidation phase during 2003-2007. Two VARs are estimated: a small VAR with spending, tax revenue, and GDP and a larger VAR which includes inflation and short term interest rate (to control for monetary conditions).

Our major findings can be summarized as follows:

- Preliminary findings for India show that discretionary fiscal policy shocks have economically significant effects on activity, with *current* government spending multiplier estimated at one (on impact), declining to around 0.5 after four to five quarters, suggesting partial crowding out of some private demand component.
- Consistent with evidence for other countries, the *development* spending multiplier is greater than 1, suggesting that composition of spending matters, with a persistent effect even at 16 quarters.
- Tax revenue multiplier is about twice as large as current spending (same order of magnitude of development spending), and remains significant after 8 quarters. This is also consistent with the cross-country evidence, which shows large tax multipliers, especially at longer horizons.

The remained of this paper is organized as follows. Section II reviews the literature on fiscal multipliers and the cross-country evidence on the impact of fiscal policy on economic activity. In this section, we also motivate the need to uncover empirical evidence on fiscal multipliers, since theoretical predictions from simple models are very sensitive to hard-to-estimate (unobserved) parameters. In other words, the quest for reliable evidence on fiscal multipliers lies in the data. Section III describes the methodology used to estimate fiscal

GDP (see <u>http://otrans.3cdn.net/45593e8ecbd339d074_I3m6bt1te.pdf</u>), researchers have rushed to publish their findings on the effects of fiscal policy on the economy. As usual, the debate has focused on advanced economies. A comprehensive survey of the pre-2008 literature is presented in Perotti (2009). Recent papers, some of which include a discussion of the current crisis-related stimulus include Barro and Redlick (2009) and Freedman *et.al.* (2009).

multipliers and Section IV presents the main empirical findings for India. Some robustness exercises are also discussed. Section V presents concluding remarks.

2. Cross-Country Evidence on Fiscal Multipliers

As we show below, tightly parameterized economic models offer limited guidance to gauge the magnitude of fiscal multipliers.² For example, in a simple flexible-price DSGE model the effect of a government spending shock on GDP depends on the elasticity of labor supply, the coefficient of relative risk aversion of the representative agent, and the share of government spending in GDP (Box 1). In a slightly more complicated model with money and price rigidity, the effect of the spending shock depends on several additional parameters, including the persistence of spending shocks.

Evidence on the effects of fiscal policy on the economy is mostly based on three approaches:³

• The *narrative* approach, pioneered by Ramey and Shapiro (1998) involves isolating the exogenous unanticipated component of fiscal policy changes and estimating reduced form regressions of GDP on dummy variables corresponding to episodes of exogenous fiscal policy changes.⁴ The event study may also focus on consumer or investment behavior, as in Shapiro and Slemrod (2003, 2009) and Barro and Redlick (2009). Evidence from such event studies is consistent with some effectiveness of fiscal policy. For instance, the 2001 income tax rebates in the United States were found to be effective in boosting consumption, but the multiplier was estimated at less than one. In the IMF's World Economic Outlook (October 2008) the results from the event studies show that the levels of public debt and composition of fiscal measures are important determinants of the effectiveness of fiscal policy; high debt

² To some extent the same criticism applies to empirical approaches, as we have argued above (and will again below) that findings generally vary with the approach used. Nonetheless, given that the size of fiscal multipliers varies not only across models but also within models (as parameters change), the assertion that the effectiveness of fiscal policy is ultimately an empirical issue remains valid.

³ A concise and insightful overview is presented in Spilimbergo *et.al.* (2009).

⁴ The dummy variables are coded based on the study of narrative accounts of policy changes in the press and official documents. The fiscal dummy variable (say, D) takes a value of one if there is a fiscal event (e.g., military buildup), and regression of the type $y_t = A(L)y_{t-1} + B(L)D_t + u_t$ are run. The cumulative impact of the fiscal even on GDP (y) is given by $B(L)(1-A(L))^{-1}$.

Box 1. Analytical Fiscal Multipliers

In a simple DSGE model with flexible prices, the representative agent maximizes:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \phi_m \frac{m_t^{1-\chi}}{1-\chi} - \phi_n \frac{N_t^{1+\xi}}{1+\xi} \right) \ , \ \sigma, \chi > 1; \xi \ge 0; \phi_m, \phi_n > 0$$

subject to the usual budget constraints:

$$C_t = \left[\int_0^1 C_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \ \theta > 1$$

and

$$\int_0^1 P_t(i)C_t(i)di + B_t + M_t \le W_t N_t + (1+i_{t-1})B_{t-1} + M_{t-1} + T_t + Q_t$$

The government's budget constraint is given by:

$$M_t - M_{t-1} + B_t = T_t + (1 + i_{t-1})B_t + P_t G_t$$

and government spending is assumed to follow the following stochastic process:

$$\begin{array}{rcl} G_t &=& \bar{G}e^{g_t} \\ g_t &=& \rho_g g_{t-1} + \epsilon^g_t \ , \ 0 < \rho_g < 1 \end{array}$$

In this model, the solution for output and inflation are given by:

$$\begin{split} \hat{y}_t &= \underbrace{\frac{s_g}{1 + \frac{\xi}{\sigma}(1 - s_g)}}_{\geq 0} g_t \\ \hat{\pi}_t &= \hat{m}_{t-1} + \underbrace{\frac{\xi \left(1 - \rho_g \beta\right)}{\chi (1 - \beta) + \beta (1 - \rho_g)} \left(\frac{s_g}{1 + \frac{\xi}{\sigma}(1 - s_g)}\right)}_{\geq 0} g_t \end{split}$$

The equation for output shows that the multiplier is positive and a function of the parameters described in the text (share of government spending, curvature of the utility function, and elasticity of labor supply).

levels lower the multiplier because of fiscal expansions are associated with rising interest rates and spreads.

- The second approach is based on full-fledged structural models. The class of models used range from the more traditional simultaneous equations models such as the one used by Macroeconomic Advisers5 to fully-optimizing DSGE models with price rigidities as in Taylor et.al. (2009). Not surprisingly, the authors find that the size of estimated multipliers is not robust. They estimate a benchmark New Keynesian DSGE models and find that multipliers are about 1/6 of the ones reported in the Romer and Bernstein (2009). Taylor et.al. (2009) also show their results are robust to the inclusion of hand-to-mouth consumers in their model, a feature that many believe is critical to generating sizable multipliers. Results based on other models in the DSGE tradition show that fiscal policy remains effective when monetary policy remains accommodative, as can be seen in the IMF analyses in Box 2.1 in the April 2008 World Economic Outlook and Freedman et.al. (2009). This point is emphasized by Christiano et.al. (2009); they find that the fiscal multiplier is large (greater than one for government spending) when the nominal interest is constant.
- The third approach has been pioneered by Blanchard and Perotti (2002). It involves identifying fiscal policy "shocks" using VARs and simulating the dynamic impact of these shocks on GDP and other variables of interest. Identification of the fiscal shocks is typically achieved by assuming that government spending is predetermined within a quarter (such assumption would not be reasonable with annual data). The VAR studies typically find a larger effect of government spending on GDP and in some cases crowding-in of consumption (e.g. Blanchard and Perotti, 2002, and Gali et.al., 2007). Other VAR studies find crowding-out of consumption and a smaller but positive effect on GDP (see Perotti, 2009). Uhlig and Mountford (2008) use less restrictive sign-restrictions to identify fiscal shocks and find much smaller deficit-spending multipliers.⁶ Interestingly, several VAR studies tend to find very large tax multipliers. This evidence is also consistent with the regression approaches of Romer and Romer (2008) and Barro and Redlick (2009), particularly at longer horizons.

⁵ This is available at <u>http://www.macroadvisers.com/content/Structure_and_Use.pdf</u>.

⁶ Sign restrictions are consistent with a broad range of models, do not require the number of shocks to equal to the number of variables, and do not impose linear restrictions on the contemporaneous relationship between reduced-form and structural shocks. Sign restrictions also have intuitive appeal. For example, the business cycle shock in Uhlig and Mountford (2008) is identified by the requirement that the impulse responses of output and taxes are positive for at least four quarters after the shock.

Assessment of impacts of Discretionary Fiscal Foncy Summus by Empirical Method							
	Output	Private Consumption	Private Investment in Durables	Private Capital Investment			
VAR studies Narrative studies Case studies	Neutral to positive Positive Positive	Neutral to positive Negative Positive	Negative to positive Negative 	Negative to positive			

Assessment of Impacts of Discretionary Fiscal Policy Stimulus by Empirical Method

Note: Studies placed in the vector autoregression (VAR) category include Fatás and Mihov (2001); Mountford and Uhlig (2002); Blanchard and Perotti (2002); and Galí, López-Salido, and Vallés (2007). Studies placed in the narrative category include Ramey and Shapiro (1998) and Edelberg, Eichenbaum, and Fisher (1999). Case studies include Johnson, Parker, and Souleles (2006).

In the case of emerging markets, the evidence is relatively limited. Ilzetzki, Vegh and Mendoza (2009) estimate fiscal multipliers for 45 countries based on the BP approach. They find that multipliers that to be larger in high income countries, in countries with predetermined exchange rates, in more closed economies, and in economies with lower debt levels. The IMF October 2008 WEO also has a detailed analysis of fiscal multipliers based on panel regressions. The results generally indicate small multipliers for both taxes and spending. The analysis in the WEO also shows that credibility of policy framework and degree of monetary accommodation is critical to the overall effectiveness of fiscal policy.

	Real GDP Response							
	Elasticity-based fise	cal impulse measure	Regression-based fiscal impulse measure					
Effect in:	Year zero	Year three	Year zero	Year three				
	(with respect to positive fiscal impulse by 1 percentage point of GDP)							
Baseline specification	0.15	-0.16	0.08	-0.02				
Country differences								
Advanced economies only	0.12	0.13	0.11	0.51				
Emerging economies only	0.21	-0.03	0.10	-0.09				
Composition								
Revenue-based policy changes	0.21	0.12	0.10	0.14				
Expenditure-based policy changes	0.13	-0.21	0.06	-0.06				
Composition: advanced economies only								
Revenue-based policy changes	0.35	0.59	0.01	0.40				
Expenditure-based policy changes	-0.09	-0.26	0.15	0.52				
Composition: emerging economies only								
Revenue-based policy changes	0.23	0.23	0.13	0.17				
Expenditure-based policy changes	0.20	-0.18	0.08	-0.23				
Downturns	0.29	0.00	0.10	0.04				
Fiscal stimulus only	-1.30	-0.88	-0.87	-0.29				
Fiscal stimulus only, high initial debt	-1.75	-2.05	-1.05	-0.80				
Fiscal stimulus only, low initial debt	-0.96	-0.36	-0.65	0.13				

Table 5.4. Responses of Real GDP to Discretionary Fiscal Policy Changes

3. Data and Methodology

Data

The data are quarterly and span the period 1996Q2-2009Q3. The variables included in the estimation are the wholesale price index (WPI), real GDP at market prices, the NEER, the 3-month nominal interbank interest rate, and foreign variables. The latter includes the "world" oil price (average from the IMF's WEO) and the 3-month LIBOR. The GDP and WPI and fiscal variables are seasonally adjusted. The fiscal variables used are based on the national accounts (in the case of government consumption) or the CGA (both current and capital spending).

Baseline VAR Identification Schemes

As note in the previous section, the VAR methodology, which has been successfully applied to identify monetary policy shocks, has been adapted by BP to simulate the effects of fiscal policy on the economy. The baseline identification assumption followed here is adapted from BP. More specifically, the VAR model utilized in this paper assumes that the model economy can be represented by:

$$y_t = c + C(L)y_t + u_t$$

where y = [G T Y] is the n x 1 data vector containing the government spending (G), tax revenues (T), and GDP (Y); c is a vector of constants, C(L) is a lag polynomial in C_k, and C_k is an n x n matrix of coefficients (with k = 1, ..., K); and u_t is a white noise vector of reducedform residuals. All variables enter the VAR in natural logarithms. In the case of the "augmented" VAR, the (natural log of) WPI and the interest rate are included. The latter is not seasonally adjusted. In the case of the reduced form VAR above, the residuals are a combination of: (i) the automatic response of government spending and taxes to GDP (automatic stabilizer effect), (ii) the *systematic* discretionary response of fiscal policy to GDP (e.g., tax cuts typically implemented during recessions), and (iii) the exogenous discretionary changes in fiscal policy, the "fiscal shocks". As can be inferred from the previous section, this last part of the residuals is what we really want to measure in order to simulate the effects of fiscal policy on the economy.

The "structure" of the economy is assumed to be linear and given by:

$$Ay_t = D(L)y_t + v_t$$

where v_t is the vector of white noise structural shocks. Without loss of generality the constant term has been omitted. The matrices A and B describe the contemporaneous relationship between the variables and the linear relationship between the structural shocks and the reduced form residuals. It is easy to show that structural shocks can be mapped from the estimated reduced residuals since Au = Bv.

In the small recursive VAR, variables are ordered from the most exogenous to the most endogenous. In our case, this corresponds to the following posited ordering: G, Y, and T. In this case: (i) government spending does not react (within a quarter) to shocks to GDP and revenues, consistent with some stickiness in spending decisions but still allows for relatively short "inside" lags; (ii) GDP reacts contemporaneously to spending shocks (but not to tax shocks); and (iii) tax revenues react to both spending and GDP shocks since revenues are assumed to be the most endogenous of the three variables included in the small VAR. In the case of the augmented VAR, the WPI is included right after GDP (but results are unchanged if the ordering with GDP is reversed) and the interest rate is ordered last. In the augmented (seven-variable) VAR with foreign variables, oil prices and the LIBOR are block exogenous.⁷

In the indentified VAR timing remains critical for the identification strategy: it is assumed that it takes longer than a quarter for discretionary fiscal policy to respond to a shock to GDP. This is equivalent to saying that the systematic discretionary response of fiscal policy is absent in quarterly data. Such an assumption is much harder to justify with annual data, but given the lags in policy implementation and budget cycles, this seems a reasonable assumption in our context. More generally, this type of timing assumption has been extensively used in the VAR literature (including on monetary policy) since it is easy to implement and it is consistent with different classes of models. In the simple VAR the relationship between the structural shocks and the reduced form residuals is given by:

[1	$-\alpha_G^Y$	$-\alpha_T^Y$	$\begin{bmatrix} u_t^Y \end{bmatrix}$	[1	0	0]	$[v_t^Y]$
$-\alpha_Y^G$	1	0	u_t^G	= 0	1	β_T^G	v_t^G
$\left[-\alpha_Y^T\right]$	0	1	$\begin{bmatrix} u_t^T \end{bmatrix}$	0	eta_G^T	1	$\left[v_{t}^{T}\right]$

with α_Y^G and β_T^G equal to zero. There are differences from the purely recursive ordering described above, and the contemporaneous restrictions imposed to identify the structural shocks can be described as follows:⁸

• As in the recursive VAR, government spending does not react contemporaneously to structural shocks to taxes and GDP (note the restrictions on α_Y^G and β_T^G); in the augmented VAR, spending responds contemporaneously to the WPI (it is imposed that government spending declines in real terms with an unanticipated increase in the WPI);

⁸ As in BP and most of the VAR literature, no restrictions are imposed on the lagged structural parameters of the model. In this case, structural spending or tax shocks can affect GDP in all periods after the initial period in which the shock occurred.

⁷ The results from the 7-variable VAR are broadly in line with those of the domestic-variables only VAR (i.e., excluding the oil price and LIBOR). However, because of substantial sampling uncertainty, the standard deviations around the impulse response functions are very wide and reliable inference is hampered.

- GDP responds contemporaneously to shocks to both fiscal variables; in the augmented VAR GDP does not react to price or interest rate shocks within the same quarter due to stickiness in production plans;
- Tax revenues react contemporaneously to both GDP and spending shocks (they are endogenous in part because they react to aggregate spending and because of the systematic discretionary component of fiscal policy discussed above); moreover, the parameter β_G^T is estimated from the data, allowing shocks to spending to affect revenue shocks—consistent with the view that revenues are determined after spending.
- The foreign interest rate and domestic output responds contemporaneously to the oil price (or commodity prices) within a quarter, but the latter is not affected by the former contemporaneously (zero restriction);
- Domestic prices respond contemporaneously to oil price shocks (in the augmented VAR) and to output (the second restriction can be relaxed without affecting the results); also, in the augmented model, the interest rate elasticity of tax revenue and government spending is set to zero, and the interest rate responds to all variables in the system.

4. Main Findings

The main results from the BP and the recursive VAR are broadly consistent with a reasonably strong effect of fiscal policy shocks on GDP. The main results from the *scaled* impulse response functions (IRFs) based on BP approach along with the IRF's 68 percent probability bands can be summarized as follows:⁹

 Current Spending multiplier is slightly above one on impact, and declines to around 0.5 after 5 quarters, suggesting a rapid crowding out of some private demand component after a couple of quarters. As in Uhlig and Mountford (2008), the deterministic component of the VAR does not include a time trend. When a time trend is included the IRF shifts



⁹ The 16 and 84 percent fractiles of the IRFs are calculated by Monte Carlo simulation as described in more detail in the Appendix.

downwards and becomes insignificant after 5 quarters. Another important result, consistent with some of the findings in Perotti (2009), is that the identified government spending shocks are fairly similar for the recursive and BP approaches. In both approaches, tax revenues increase on impact by less than the increase in spending, suggesting that the "pure" can be identified as a deficit-financed spending shock. In the case of the sign-restriction approach, tax revenue does not increase at all in the first 4 quarters. Inflation increases gradually with the spending shock, with the effect peaking after 6 quarters. Interest rates also go up, but the effect is not significant, suggesting that crowding out operates through some other channel.

• Development spending multiplier is greater than 1, suggesting that composition of

spending matters (consistent with cross-country evidence discussed above). The uncertainty surrounding this multiplier is also large, probably reflecting the volatility of development spending. Interestingly, the effect persists even after 16 quarters, suggesting some crowding in effects of that type of spending.



Tax revenue multiplier is about twice as large as current spending (same order of magnitude of development spending), and remains significant after 8 quarters.

Given that spending does not react to the tax shock, the experiment can be interpreted as a deficitreducing tax increase. The result is broadly in line with Uhlig and Mountford (2008) and Romer and Romer (2008), which report very large tax multipliers. The former argue that the distortionary effect of taxes



shows up at longer horizons, underscoring the need for proper dynamic scoring of tax cuts. Another interesting result concerning the tax shock is that the immediate effect of the tax increase on GDP is the same as in recursive approach (when it is restricted to zero). In the sign-restriction approach tax increases lower GDP on impact.

Remarks on Crowding Out

The results above suggest that crowding out might dull the effects of fiscal policy. First, the current spending multiplier is well below one after a few quarters, suggesting that the increased spending reduces the availability of resources for the private sector, leading to crowding out. The effect on growth over the longer term depends on which component of demand declines, but given the relatively small size of the multiplier and the evidence (in other countries) that consumption of credit-constrained households is not very sensitive to interest rates, it is likely that private investment declines following a deficit-financed increase in government spending. Moreover, in India as in other developed and emerging economies, higher deficits are not always accompanied by higher interest rates. In the case of our model, the estimations were conducted with short term interest rates. But as seen recently, long term interest rates (which are more relevant for investment decisions) have displayed sensitivity to budget announcements. Thus the existence of a traditional crowding out effect with higher long term interest rates causing a decline in private investment cannot be ruled out.

The credibility of the fiscal policy and the fiscal framework more generally are also important determinants of the effectiveness of fiscal policy. While interest rates on government bonds may not respond to bad news about the fiscal position, credit spreads may do the job, raising the cost of financing for corporates and households. Agca and Celasun (2009) find that public external debt has a sizable positive impact on corporate syndicated loan spreads. Their findings are consistent with the view that fiscal expansions and the associated debt buildup may crowd out private *access* to external markets by increasing spreads. In the case of India since they also show that while increases in *overall* public debt raises private borrowing costs in external markets, but the main driver of this relationship is *external* public debt.

Additional Evidence from Indian States

Data for the states can also be used to estimate fiscal expenditure multipliers. The evidence above suggests that spending multiplier is around one, broadly consistent with the finding above. The estimation of the states' spending multiplier follows the cross-country empirical literature (WEO, 2008 and Gupta et al. (2004). While it is hard to find credible instruments for government spending (such as election cycles), GMM dynamic panel estimation is applied to identify the causal impact of spending on economic activity. The results are presented in the table below. As can be seen from the table, the estimated multipliers range from 0.9 to 1.3, suggesting some crowding-out of private demand.

	Log (real per capita GSDP)				Growth (Real per capita GSDP)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log (real per capita primary spending)	0.206**	0.222***	0.158**					
	[0.093]	[0.080]	[0.070]					
Growth (real per capita primary spending)				0.175	0.189*	0.165	0.187	0.187*
				[0.109]	[0.112]	[0.111]	[0.115]	[0.113]
Ν	313	313	312	312	298	312	298	283
Lag Dependent Variable	yes	yes	yes	yes	yes	yes	yes	yes
Lag Spending	yes	yes	no	no	yes	no	yes	yes
State Fixed Effect	yes	yes	no	no	no	yes	yes	no
Year Fixed Effect	yes	yes	yes	yes	yes	yes	yes	yes
State time trend	no	yes	no	no	no	no	no	no
XTABOND	no	no	yes	no	no	no	no	yes
Implied multiplier	1.2	1.3	0.9	1.1	1.1	1.0	1.1	1.1

5. Concluding Remarks

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This paper assesses the effects of fiscal policy on economic activity in India over the last decade and half and finds that fiscal policy can play an effective countercyclical role. The results also have implications for the design of fiscal consolidation plans going forward. In particular, our finding suggest that expenditure reform aimed at curtailing the growth of spending may be preferable to tax increases because the latter may have larger (negative) effects on growth over the longer term.

The findings also shed light on the nature of crowding out and the need for careful dynamic scoring of fiscal plans. The inclusion of debt in the empirical models and further analysis of the effects on fiscal shocks and *announced* fiscal measures on aggregate demand components are important issues for future research.

Appendix I. Estimation Details

The reduced form model is estimated with four lags in log-levels, except for the domestic and foreign interest rate.¹⁰ A reduced form with 4 lags *and a time trend* also yields a good fit, with the reduced form passing the standard specification tests for autocorrelation and heteroskedasticity. Regarding the normality of the residuals, there is some excess kurtosis as indicated by the Jarque-Bera test. While all variables can be characterized as nonstationary (or near-nonstationary as in the case of interest rates) according to standard unit roots tests, most findings are robust to first differencing and inference can still be conducted with the estimated model in levels (se for example Canova, 2007, page 125). The structural model can be rewritten in reduced form as:

$$y_t = c + C_1 y_{t-1} + \dots + C_p y_{t-p} + e_t$$

where $e_t = B_0^{-1}u_t$, with variance-covariance matrix given by $\Omega = B_0^{-1}D(B_0^{-1})'$, where D is the variance-covariance matrix of the structural shocks. The matrix Ω can be rewritten as $\Omega = ADA'$ where D is diagonal. In this case, since $u_t = A^{-1}e_t$, with $A = B_0^{-1}$, then $E(u_tu_t') = E(A^{-1}e_te_t'(A^{-1})') = A^{-1}(ADA')(A^{-1})' = D$, and the vector u_t can be interpreted as "structural" shocks.¹¹ Identification amounts to imposing restrictions on the matrix $A(B_0^{-1})$ that orthogonalizes the reduced form errors, eliminating their contemporaneous correlation.¹² A widely-used identification scheme is the recursive ordering (Cholesky), which assumes that A has a lower triangular structure. This is equivalent to a hierarchical ordering of the variables, with the most exogenous variable ordered first. Statistical inference can be conducted directly based on the estimated log-likelihood. If there are n* estimated parameters in B0, the number of over-identifying restrictions (r) is given by $r = (n(n-1)/2) - n^*$. The test for over-identifying restrictions is based on the maximized value of the log-likelihood and has a chi-square distribution with r degrees of freedom.¹³

¹¹ Since $e = B_0^{-1}u$ and $u = A^{-1}e$, the equality $A = B_0^{-1}$ follows immediately.

¹² Alternatively, note that the matrices B_0 and D cannot have more unknowns than Ω . In this case, since D has n parameters (it is diagonal) and Ω has n(n+1)/2 parameters (it is symmetric), this constrains B_0 to have at most n(n-1)/2 free parameters.

¹³ The standard errors of the impulse responses are calculated by Monte Carlo simulation. They are broadly similar to the probability bands are calculated from a Bayesian method that employs a Gaussian approximation to the posterior of the matrix A (recommended by Sims and Zha (1999) for overidentified models).

¹⁰ The structural parameters are estimated by maximum likelihood, but it may also be estimated by solving the nonlinear system given by $\Omega = B_0^{-1}D(B_0^{-1})'$

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